

Towards the field theory of the Standard Model on fractional D6-branes on T^6/\mathbb{Z}'_6 : Yukawa couplings and masses

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We present the perturbative Yukawa couplings of the Standard Model on fractional intersecting D6-branes on T^6/\mathbb{Z}'_6 and discuss two mechanisms of creating mass terms for the vector-like particles in the matter spectrum, through perturbative three-point couplings and through continuous D6-brane displacements.

1 Introduction

While the massless spectra of D6-brane models in type IIA string theory have been discussed in a variety of cases, see e.g. the statistics for various orbifold backgrounds in [1, 2, 3], the inspection of the low-energy field theory limit has to a large extent focussed on the gauge couplings, see e.g. [4, 5, 6] and [7] for a discussion within the string landscape. For interaction terms such as Yukawa couplings, exact string theoretic results are only known on the six-torus, e.g. [8, 9], and its $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold without discrete torsion, and consequently a treatment of Standard Model (SM) vacua in globally consistent supersymmetric D6-brane models on T^6/\mathbb{Z}'_6 [10, 11, 12, 13, 3, 7] or other T^6/\mathbb{Z}_{2N} [14, 15, 16] and $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2M}$ backgrounds with discrete torsion [17, 18] has not been performed. To go one step further in the understanding of the field theory of intersecting *fractional* D6-brane SM vacua, we assume here that the formula for Yukawa couplings on the six-torus directly carries over to its orbifolds and extends to the new case with chiral matter at one vanishing angle, i.e. ‘triangular’ worldsheets with an acute angle zero. Our discussion focusses on the dominant worldsheet contribution to each three-point coupling.

1.1 Geometry of a D6-brane Standard Model vacuum with ‘hidden’ $USp(6)$

The D6-brane configuration of a particular SM vacuum on the **ABa** lattice orientation of $T^6/(\mathbb{Z}'_6 \times \Omega\mathcal{R})$, which is generated by $\theta : z_k \rightarrow e^{2\pi i v_k} z_k$ with $\vec{v} = \frac{1}{6}(1, 2, -3)$ and $\mathcal{R} : z_k \rightarrow \bar{z}_k$, is given in table 1. The resulting massless open string spectrum consists of a ‘chiral’ part computed from intersection numbers (i.e. including the anomalous $U(1)_b \subset U(2)_b$ charge of otherwise vector-like matter), which contains the SM quarks and leptons and nine Higgs generations [3, 7],

$$\begin{aligned}
 [C] &= 3 \times \left[(\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + (\bar{\mathbf{3}}, \mathbf{1})_{\frac{-2}{3}} + (\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_0 + 2 \times (\mathbf{1}, \mathbf{2})_{\frac{-1}{2}} + (\mathbf{1}, \mathbf{2})_{\frac{1}{2}} \right] + 9 \times \left[(\mathbf{1}, \bar{\mathbf{2}})_{\frac{-1}{2}} + (\mathbf{1}, \bar{\mathbf{2}})_{\frac{1}{2}} \right] \\
 &\equiv 3 \times [Q_L + d_R + u_R + e_R + \nu_R + 2 \times L + \bar{L}] + 9 \times [H_d + H_u],
 \end{aligned} \tag{1}$$

where we already used the breaking $USp(2)_c \rightarrow U(1)_c$ by a continuous displacement σ_c^2 . Besides the lower case index of the hyper charge, $Q_Y = \frac{Q_a}{6} + \frac{Q_c}{2} + \frac{Q_d}{2}$, also the baryon minus lepton number

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D6-branes for the Standard Model with $USp(6)_h$ on T^6/\mathbb{Z}'_6				
D6-brane	label	angle on $(T^2_{(1)}, T^2_{(2)}, T^2_{(3)})$ w.r.t. $\Omega\mathcal{R}$ -plane	displacement (σ^1, σ^3) on $(T^2_{(1)}, T^2_{(3)})$	orientation on $T^2_{(2)}$ vs. $\Omega\mathcal{R}$ -plane
Baryonic	a	$\pi(-\frac{1}{3}, -\frac{1}{6}, \frac{1}{2})$	$(1; 1)$	
Left	b	$\pi(\frac{1}{6}, -\frac{1}{3}, \frac{1}{6})$	$(1; 0)$	
Right	c	$\pi(-\frac{1}{3}, \frac{1}{3}, 0)$	$(1; 1)$	\parallel
Leptonic	d	$\pi(\frac{1}{6}, \frac{1}{3}, -\frac{1}{2})$	$(0; 1)$	
Hidden	h	$\pi(-\frac{1}{3}, -\frac{1}{6}, \frac{1}{2})$	$(0; 0)$	\perp
				$USp(6)_h$

Table 1 Five supersymmetric stacks of D6-branes, which cancel all RR tadpoles on the **ABa** lattice of T^6/\mathbb{Z}'_6 with SM spectrum and a ‘hidden’ $USp(6)_h$ gauge factor. The enhancements $U(N) \rightarrow USp(2N)$ arise for D6-branes parallel or perpendicular to the O6-planes. Only in the first case, the enhancement can be cancelled by a continuous displacement σ^2 .

Multiplicities and localisations of leptons					
Particle	$x(\theta^k y)$	$\chi/\varphi_{x(\theta^k y)}$	$T^2_{(1)}$	$T^2_{(2)}$	$T^2_{(3)}$
$L_{1\dots 6}$	bd	\emptyset	$\parallel \neq$	1, 2, 3	3
	$b(\theta d)$	-6	4, (R, R')	1, 2, 3	3
	$b(\theta^2 d)$	$ 4 _m$	5, (S, S')	$\parallel =$	3
$\bar{L}_{1\dots 3}$	bd'	$ 2 _m$	5, (S, S')	$\parallel =$	3
	$b(\theta d')$	\emptyset	$\parallel \neq$	1, 2, 3	3
	$b(\theta^2 d')$	-3	4, (R, R')	1, 2, 3	3
$E_{1\dots 3}$	cd	$ 2 _m$	(T, T')	$\parallel =$	3
	$c(\theta d)$	3	4	1, 2, 3	3
	$c(\theta^2 d)$	\emptyset	5	1, 2, 3	3

Table 2 Assignments of left- and right-handed leptons to a given intersection sector $x(\theta^k y)$ with counting of chiralities $\chi_{x(\theta^k y)}$ or multiplicities $\varphi_{x(\theta^k y)}$ of the non-chiral matter content $\varphi_{x(\theta^k y)}$. The notation $E_i = (e^i_R, \nu^i_R)$ is tailored for the right-symmetric case $USp(2)_c$ with $\sigma^2 = 0$. Family replication in the lepton sector is due to three intersection points 1,2,3 on $T^2_{(2)}$. The quarks display a more intricate pattern on $T^2_{(1)} \times T^2_{(2)}$.

Multiplicities and localisations of quarks & Higgses					
Particle	$x(\theta^k y)$	$\chi/\varphi_{x(\theta^k y)}$	$T^2_{(1)}$	$T^2_{(2)}$	$T^2_{(3)}$
Q_1, Q_2	ab'	-2	4	1, 4	3
Q_3, Q_4	$a(\theta b')$	-2	4, 5	1	3
\bar{Q}	$a(\theta^2 b')$	1	5	1	3
U_1, U_2	ac	2	$\parallel =$	1, 4	3
	$a(\theta c)$	\emptyset	5	1	3
U_3	$a(\theta^2 c)$	1	4	1	3
$H_{1\dots 6}$	bc	6	4, 5	1, 2, 3	3
$H_{7\dots 9}$	$b(\theta c)$	3	5	1, 2, 3	3
	$b(\theta^2 c)$	$ 2 _m$	4	$\parallel =$	3

symmetry, $Q_{B-L} = \frac{Q_a}{3} + Q_d$, is gauged and anomaly-free in this model. The ‘vector-like’ spectrum from vanishing intersection numbers consists of two parts, the first with SM charges only,

$$\begin{aligned}
[V] = & \left[(\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + 3 \times (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + 3 \times (\bar{\mathbf{3}}, \mathbf{1})_{\frac{-2}{3}} + 3 \times (\bar{\mathbf{3}}_{\text{Anti}}, \mathbf{1})_{\frac{1}{3}} + 6 \times (\mathbf{1}, \mathbf{3}_{\text{Sym}})_0 + c.c. \right] \\
& + \left[1_m \times (\mathbf{1}, \bar{\mathbf{2}})_{\frac{-1}{2}} + 1_m \times (\mathbf{1}, \bar{\mathbf{2}})_{\frac{1}{2}} + 2_m \times (\mathbf{1}, \mathbf{2})_{\frac{-1}{2}} + 1_m \times (\mathbf{1}, \mathbf{2})_{\frac{1}{2}} + 4_m \times (\mathbf{1}, \mathbf{1}_{\text{Anti}})_0 + c.c. \right] \\
& + \left[2_m \times (\mathbf{1}, \mathbf{1})_1 + 1_m \times (\mathbf{1}, \mathbf{1})_0 + c.c. \right] + 2 \times (\mathbf{8}_{\text{Adj}}, \mathbf{1})_0 + 10 \times (\mathbf{1}, \mathbf{3}_{\text{Adj}})_0 + 26 \times (\mathbf{1}, \mathbf{1})_0, \quad (2)
\end{aligned}$$

and the second consisting of massless states with charges under the ‘hidden’ gauge group $USp(6)_h$,

$$[H] = 2 \times (\mathbf{1}, \mathbf{1}; \mathbf{15}_{\text{Anti}})_0 + (\mathbf{1}, \mathbf{2}; \mathbf{6})_0 + (\mathbf{1}, \bar{\mathbf{2}}; \mathbf{6})_0 + 2 \times \left[(\mathbf{1}, \mathbf{1}; \mathbf{6})_{\frac{1}{2}} + (\mathbf{1}, \mathbf{1}; \mathbf{6})_{\frac{-1}{2}} \right]. \quad (3)$$

To investigate the interactions among the massless open string states, it is necessary to know their origin from a given intersection sector $x(\theta^k y)_{k \in \{0,1,2\}}$ of a D6-brane x with the orbifold images $(\theta^k y)$ of y , which can e.g. be computed using the beta function coefficients of gauge threshold amplitudes [7]. In addition, the localisations of the intersection points along T^6/\mathbb{Z}'_6 are needed. As examples, the localisations of leptons and of quarks and Higgses are displayed in table 2 and 3, respectively (for more details see [19]). The situation is depicted in figure 1 for D6-branes b, c and (θd) supporting the left-handed leptons L_i , right-handed leptons E_j and six of the Higgs generations H_k .

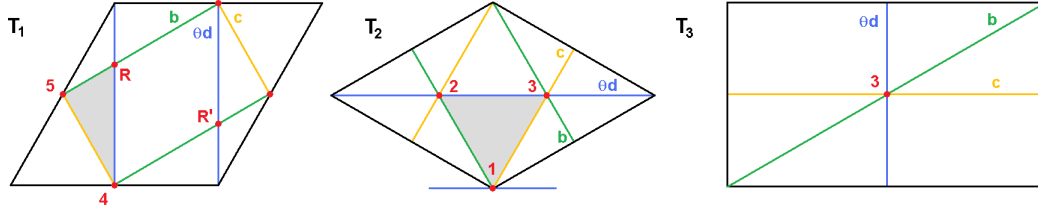


Fig. 1 The D6-branes b (green), c (yellow) and (θd) (blue) on the **ABa** lattice on T^6/\mathbb{Z}_6' . Family replication in the lepton sector is due to the intersections 1,2,3 on the two-torus $T_{(2)}^2$ without \mathbb{Z}_2 action. The two pairs $(4, R \xleftrightarrow{\mathbb{Z}_2} R')$ and $(4,5)$ of \mathbb{Z}_2 invariant intersection orbits along $T_{(1)}^2$ double the respective multiplicities of left-handed leptons L_i at $b(\theta d)$ intersections and of Higgses H_k at bc intersections.

2 Yukawa couplings

2.1 Holomorphic three-point couplings

Three-point couplings arise in perturbation theory from triangular worldsheets with edges given by the three D6-branes under which matter is charged. For example, three chiral multiplets ϕ_{xy}^i , ϕ_{yz}^j and ϕ_{zx}^k with respective charges $(\overline{\mathbf{N}}_x, \mathbf{N}_y)$, $(\overline{\mathbf{N}}_y, \mathbf{N}_z)$ and $(\overline{\mathbf{N}}_z, \mathbf{N}_x)$ under $U(N_x) \times U(N_y) \times U(N_z)$ lead to a superpotential term of the form

$$W = W_{ijk} \phi_{xy}^i \phi_{yz}^j \phi_{zx}^k \quad \text{with} \quad W_{ijk} = \prod_{m=1}^3 \left(\sum_{\mathcal{A}_{ijk,(m)}} e^{-\mathcal{A}_{ijk,(m)}} \right). \quad (4)$$

Besides the field theoretic consistency of charge neutrality, the string theoretic selection rule states that $W_{ijk} \neq 0$ is only possible if x, y and z form a closed triangle with area $\mathcal{A}_{ijk,(m)}$ along $T_{(m)}^2$, which can also be shrunk to a point or, for *fractional* or *rigid* D6-branes, have one apex with vanishing angle. The infinite sums in (4) are due to the periodicity of the underlying six-torus. If e.g. only the two-torus $T_{(2)}^2$ is considered with vanishing Wilson lines $\tau_x^2 \equiv 0$ for all D6-branes $x \in \{a, b, c, d, h\}$, the sum over areas can be compactly written as [8, 9],

$$\vartheta \left[\begin{smallmatrix} \delta_2 \\ 0 \end{smallmatrix} \right] \left(\frac{t_2 \mathcal{A}_{(2)}}{2\pi} \right) = \sum_{l \in \mathbb{Z}} e^{-t_2 \mathcal{A}_{(2)} (\delta_2 + l)^2} \quad \delta_2 = 0, t_2 = 3 \quad 1 + 2e^{-3\mathcal{A}_{(2)}} + 2e^{-12\mathcal{A}_{(2)}} + \dots, \quad (5)$$

with $t_2 \in \mathbb{N}$ the (absolute value of the) product of torus intersection numbers, δ_2 a linear function of the location of intersections and the corresponding displacements σ_x^2 and $\mathcal{A}_{(2)} > 1$ the two-torus volume in units of $2\pi\alpha'$. The sum in (5) is dominated by the term $l = 0$, as can be seen in the example of the family diagonal lepton Yukawa couplings at b, c and (θd) intersections with $\delta_2 = 0$ and $t_2 = 3$. The sum on the r.h.s. of (5) converges very fast since already the first non-trivial contribution $2e^{-3\mathcal{A}_{(2)}} < 2e^{-3} < 0.1$ is tiny for areas $\mathcal{A}_{(2)} > 1$ in the geometric regime of type IIA string compactifications. Non-diagonal matter couplings arise typically from worldsheets with a non-vanishing triangular area, which can be parameterised as a fraction $\delta_2 \in \mathbb{Q}$ of the two-torus area $\mathcal{A}_{(2)}$. Typical numerical examples of such suppression factors are given in table 4. A hierarchical structure of Yukawa couplings for lepton and quark families can thus be generated by a suitable choice of matter localisations and corresponding triangular areas. Two ways of tuning arise by varying the ratios $\mathcal{A}_{(i)}/\mathcal{A}_{(j)}$ of two-torus volumes ($i \neq j$) and by choosing different continuous displacement parameters σ_x^2 for each D6-brane x along $T_{(2)}^2$ as displayed in figure 2.

Suppression factors of pert. 3-point couplings by worldsheet areas									
\mathcal{A}	1	2	3	5	10	20	30	50	100
$e^{-\mathcal{A}/12}$	0.920	0.846	0.779	0.659	0.435	0.189	0.082	0.016	$2 \cdot 10^{-4}$
$e^{-\mathcal{A}/6}$	0.846	0.717	0.607	0.435	0.189	0.036	0.007	$2 \cdot 10^{-4}$	$6 \cdot 10^{-8}$
$e^{-\mathcal{A}/4}$	0.779	0.607	0.472	0.287	0.082	0.007	$6 \cdot 10^{-4}$	$4 \cdot 10^{-6}$	10^{-11}
$e^{-\mathcal{A}/3}$	0.717	0.513	0.368	0.189	0.036	0.001	$5 \cdot 10^{-5}$	$6 \cdot 10^{-8}$	$3 \cdot 10^{-15}$
$\mathcal{A}^{-1/4}$	1	0.841	0.760	0.669	0.562	0.473	0.427	0.376	0.316

Table 4 Numerical values of area suppressed Yukawa couplings for various sizes of the two-torus area \mathcal{A} in units of $2\pi\alpha'$. The larger the area, the better the approximation by the first contribution to the family diagonal couplings.

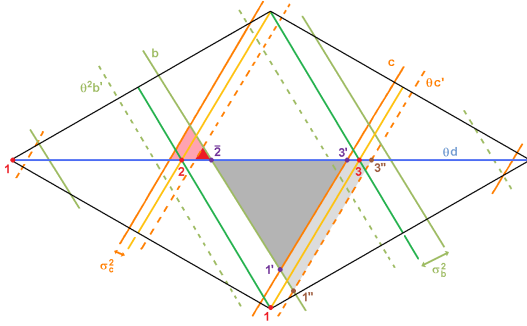


Fig. 2 Generation of Yukawa hierarchies within one generation (small red triangles) and in the flavour-mixing sectors (large grey triangles). In comparison to the original D6-brane configuration of figure 1, upon breaking $USp(2)_c \rightarrow U(1)_c$ by setting $\sigma_c^2 \neq 0$, the coupling $e_R^2 L_2 H_d^2$ receives a suppression by a smaller area than the coupling $\nu_R^2 L_2 H_u^2$ depicted in dark and dark+light red, respectively. The change of area in the non-diagonal couplings acts in the opposite way as depicted in light+dark and dark grey, respectively, for the $e_R^1 L_2 H_d^3$ and $\nu_R^1 L_2 H_u^3$ examples.

2.2 Physical Yukawa couplings

The couplings in the holomorphic superpotential (4) are in the supergravity theory dressed by a non-holomorphic prefactor involving the Kähler potential \mathcal{K} and its derivatives, the Kähler metrics K_{xy} ,

$$Y_{ijk} = (K_{xy} K_{yz} K_{zx})^{-1/2} e^{\kappa_4^2 \mathcal{K}/2} W_{ijk}. \quad (6)$$

While the Kähler potential \mathcal{K} in the exponential is universal for all three-point interactions, the Kähler metrics $K_{\mathbf{R}_a} = (f(S, U) / \prod_{i=1}^3 \sqrt{\mathcal{A}_{(i)}}) F(\vec{\phi}_{ab})$ with $f(S, U) = (S \prod_{i=1}^{h_{21}^{\text{bulk}}} U_i)^{-\alpha/4}$ and $(h_{21}, \alpha) = (1, 2)$ for T^6/\mathbb{Z}'_6 depend on the intersection angles of the corresponding D6-branes through $F(\vec{\phi}_{ab}) = \prod_{i=1}^3 \sqrt{\Gamma(|\phi_{ab}^{(i)}|) / \Gamma(1 - |\phi_{ab}^{(i)}|)}^{-\text{sgn}(\phi_{ab}^{(i)}) / \text{sgn}(I_{ab})}$ or $F(0^{(i)}, \phi_{ab}^{(j)}, \phi_{ab}^{(k)}) = \sqrt{2\pi \mathcal{A}_{(i)} V_{ab}^{(i)}}$ and $V_{ab}^{(i)}$ the normalised one-cycle volume of D6-branes a, b parallel along $T_{(i)}^2$ [20, 21]. The non-holomorphic prefactor in (6) provides thus a second potential source for the generation of Yukawa hierarchies through the functions $F(\vec{\phi}_{ab})$ [19], namely either different choices of supersymmetric angle configurations ($\vec{\phi}_{ab}$) or the normalised one-cycle volume $V_{ab}^{(i)}$ times two-torus area $\mathcal{A}_{(i)}$.

2.3 Yukawa couplings for the Standard Model with $USp(6)_h$ on T^6/\mathbb{Z}'_6

The holomorphic Yukawa interactions for the D6-brane configuration in table 1 can be derived from the localisations of SM matter states in table 2. Since all left-handed leptons L_i arise at $b(\theta d)$ intersections and all right-handed leptons E_j at $c(\theta d)$ intersections, closed triangular worldsheets are only spanned if the third apex is formed by one of the bc intersections, at which six of the Higgs families H_k are localised. The dominant terms provide diagonal Yukawa couplings with one Higgs generation per lepton family, whereas all area suppressed couplings providing lepton flavour mixing couplings,

$$\left. \begin{aligned} W_{E_i L_i H_i} &\sim \mathcal{O}(1) \\ W_{E_i L_{i+3} H_{i+3}} &\sim \mathcal{O}(e^{-\mathcal{A}_{(1)}/12}) \\ W_{E_i L_{i+3} H_{i+3}} &\sim \mathcal{O}(e^{-\mathcal{A}_{(1)}/12}) \end{aligned} \right\} \quad \text{with} \quad i = 1, 2, 3,$$

$$W_{E_i L_j H_k} \sim \mathcal{O}(e^{-\mathcal{A}_{(2)}/6}) \quad \text{with} \quad (i, j, k) \text{ permutations of } (1, 2, 3),$$

$$\left. \begin{aligned} W_{E_i L_j H_k} &\sim \mathcal{O}(e^{-\mathcal{A}_{(1)}/12 - \mathcal{A}_{(2)}/6}) \\ W_{E_i L_j H_{k-3}} &\sim \mathcal{O}(e^{-\mathcal{A}_{(1)}/3 - \mathcal{A}_{(2)}/6}) \end{aligned} \right\} \quad \text{with} \quad \left\{ \begin{array}{ll} i=1 & (j,k) = (5,6), (6,5) \\ 2 & (6,4), (4,6) \\ 3 & (4,5), (5,4) \end{array} \right.$$

Since all worldsheets are spanned by triangles with edges b , c and (θd) , the non-holomorphic prefactor $(K_{bc} K_{c(\theta d)} K_{b(\theta d)})^{-1/2} = g(S, U, \mathcal{A}) \frac{1}{2(50)^{1/4}}$ with $g(S, U, \mathcal{A}) \equiv \frac{(\mathcal{A}_{(1)} \mathcal{A}_{(2)} \mathcal{A}_{(3)})^{3/4}}{f(S, U)^{3/2}}$ and $\frac{1}{2(50)^{1/4}} \approx 0.188$ is universal in the lepton sector.

The quark Yukawa couplings display a different pattern since left- and right-handed quarks arise each in two sectors, ab' and $a(\theta^2 b')$ and ac and $a(\theta^2 c)$, respectively. The ac sector is exemplary for the exceptional situation that on *fractional* D6-branes chiral matter states can arise at one vanishing intersection angle along $T_{(1)}^2$. We therefore include the option of a triangle with zero angle and vanishing area, for which the infinite sum formula (5) may require modification. The dominant Yukawa interactions involve only two right- and three left-handed quark generations U_1, U_3 and Q_1, Q_3, Q_4 plus three Higgs generations H_1, H_4, H_7 , one of which also couples to the first lepton generation,

$$W_{U_i Q_j H_k} \sim \left\{ \begin{array}{ll} \mathcal{O}(1) & \text{with } (i, j, k) = (3, 1, 1), (3, 4, 7), (1, 3, 1), (1, 4, 4), \\ \mathcal{O}(e^{-\mathcal{A}_{(1)}/4}) & (3, 1, 4), (3, 3, 7), \\ \mathcal{O}(e^{-\mathcal{A}_{(2)}/12}) & (3, 2, l), (2, 3, 5-l), (2, 4, 8-l), \\ \mathcal{O}(e^{-\mathcal{A}_{(2)}/6}) & (3, 4, 6+l), \\ \mathcal{O}(e^{-\mathcal{A}_{(1)}/4 - \mathcal{A}_{(2)}/12}) & (3, 2, 3+l), \\ \mathcal{O}(e^{-\mathcal{A}_{(1)}/4 - \mathcal{A}_{(2)}/6}) & (3, 3, 6+l). \end{array} \right. \quad \text{with } l \in \{2, 3\}.$$

Since quark Yukawa couplings arise from three different types of triangular worldsheets, distinct non-holomorphic prefactors arise, $(K_{ab'} K_{a(\theta^2 c)} K_{bc})^{-1/2} = g(S, U, \mathcal{A}) \frac{1}{10^{3/4}}$ with $\frac{1}{10^{3/4}} \approx 0.178$, $(K_{a(\theta b')} K_{a(\theta^2 c)} K_{b(\theta^2 c)})^{-1/2} = g(S, U, \mathcal{A}) \frac{1}{5 \cdot 2^{1/4}}$ with $\frac{1}{5 \cdot 2^{1/4}} \approx 0.168$ and $(K_{a(\theta b')} K_{ac} K_{b(\theta^2 c)})^{-1/2} = g(S, U, \mathcal{A}) \frac{3^{1/8}}{2(25\pi v_1)^{1/4}}$ with $\frac{3^{1/8}}{2(25\pi v_1)^{1/4}} \approx \frac{0.193}{\mathcal{A}_{(1)}^{1/4}}$, which, however, are numerically all of the same order of magnitude as the one for the leptons, cf. also the last line of table 4 for the very mild additional suppression by $\mathcal{A}^{-1/4}$ in the last case.

2.4 Masses for vector-like matter

The vector-like matter spectrum can be decomposed into three types with different string theoretic origin and consequently distinct mechanisms by which they acquire masses [19]. The first kind consists of $\mathcal{N} = 2$ supersymmetric sectors with two D6-branes parallel along $T_{(2)}^2$, for which masses are provided by separations of the D6-branes. The matter states in brackets on the second and third line of equation (2) are of this type with $\sigma_b^2 \neq 0$ and $\sigma_c^2 \neq 0$, respectively, providing the masses.

A second type of vector-like matter arises at two distinct $\mathcal{N} = 1$ supersymmetric intersections such as the left-handed quarks Q_i at $a(\theta^k b')_{k \in \{0,1\}}$ and the conjugate \bar{Q} at an $a(\theta^2 b')$ intersection in table 3. The masses arise via couplings to SM singlets, e.g. those inside the adjoint representations A_i of $U(3)_a$, $(\mathbf{9}_{U(3)_a}) = (\mathbf{8}_{SU(3)_a})_0 + (\mathbf{1})_0$,

$$W_{\bar{Q} Q_i A_2} \sim \left\{ \begin{array}{ll} \mathcal{O}(1) & i = 4 \\ (e^{-\mathcal{A}_{(1)}/8}) & 1 \\ (e^{-\mathcal{A}_{(1)}/4}) & 3 \\ (e^{-\mathcal{A}_{(1)}/8 - \mathcal{A}_{(2)}/4}) & 2 \end{array} \right. , \quad W_{\bar{W}_{\bar{i}} W_j A_2} \sim \left\{ \begin{array}{ll} \mathcal{O}(1) & \bar{i}, j \in \{2, 3\} \\ (e^{-\mathcal{A}_{(1)}/8}) & (\bar{i}, j) = (1, 1) \end{array} \right. , \quad (7)$$

which provide mass terms if A_2 receives a *vev* along some flat direction of the (trivial D-term) and F-term potential, see [19] for more details. The example on the r.h.s. shows the same mechanism for the

$(\bar{\mathbf{3}}_{\text{Anti}}, \mathbf{1})_{1/3}$ representations W_j and their conjugates $\bar{W}_{\bar{j}}$. By combining several *vevs* of singlets inside the adjoints of $U(3)_a$ and $U(2)_b$, all states on the first line and the remaining charged states on the last line of (2) are rendered massive. The matter states with $SU(2)_b \times U(1)_Y$ charge in the ‘hidden’ spectrum (3) acquire masses by a similar mechanism, namely if the $(\mathbf{15}_{\text{Anti}})_0$ of $USp(6)_h$ receives some *vev*.

The last type of vector-like matter consists of three lepton (L_{3+i}, \bar{L}_i) and nine Higgs (H_u^i, H_d^i) pairs in equation (1), which are ‘chiral’ w.r.t. the anomalous $U(1)_b$ symmetry. These states couple perturbatively to the symmetric representations \bar{b}_i of $U(2)_b$ or its conjugate b_i instead of the adjoints for the above mentioned vector-like quark pairs, e.g. the non-suppressed couplings read $W_{\bar{L}_i b_i L_{3+i}}, W_{H_i \bar{b}_i H_j} \sim \mathcal{O}(1)$. Any *vev* will then lead to a gauge symmetry breaking $SU(2)_b \rightarrow U(1)_{I_3}$, which might happen at an intermediary energy scale between M_{weak} and M_{string} [19]. Alternatively, D-brane instantons or higher order n -point couplings might provide mass terms without gauge symmetry breaking.

3 Conclusions

We presented an estimation of the relative order of magnitude of Yukawa couplings in the lepton and quark sector for the SM on T^6/\mathbb{Z}'_6 based on a scrutiny of the worldsheets with minimal areas. While the dominant terms for the leptons are flavour diagonal, mixings already occur at leading order in the quark sector. Our derivation of perturbative three-point couplings, which also includes masses for the vector-like matter states through Higgs-like couplings to Standard Model singlets, relies on methods developed for the six-torus. A thorough string theoretic derivation is needed to clarify if further stringy selection rules such as discrete symmetries restrict the sums over worldsheet areas and if a similar lattice sum occurs as well for the special case of one vanishing angle. Last but not least, higher order and D-instanton couplings are expected to modify the subleading behaviour.

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